Parabolic plasma sheath potentials and their implications for the charge on levitated dust particles

E. B. Tomme*, D. A. Law, B. M. Annaratone, and J. E. Allen

Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, United Kingdom

Analysis of several numerical plasma sheath models as well as data from several previously reported experiments have indicated that the sheath potential function may be very closely approximated with a parabola. In this letter, we demonstrate that once this potential function is suitably determined the charge on dust particles levitated in the plasma sheath may be calculated directly from their equilibrium heights.

Plasmas are commonly contaminated by dust. For example, ionospheric dust complicates interpretation of over-the-horizon radar returns [1] while dust from volcanic emissions and other sources form rings around several planets in the solar system [2]. While exact yield numbers are closely held industrial secrets, it is known that dust contamination in semiconductor processing plasmas causes a large portion of the product to be discarded as waste [3]. Dust purposely grown in plasmas can also be used in a variety of positive ways. For example, the deposition of dust during the manufacture of nanostructured silicon can accelerate the crystallisation process [4], lower the number of crystal defects [5] and lead to the production of more efficient solar cells [6]. The charge on dust suspended in these plasmas is a critical but not well-understood parameter for eliminating or efficiently using the dust.

The subject of the plasma sheath is also of wide interest. As the ions required for etching of semiconductors receive their energy from the fields within the sheath, knowledge of the spatial distribution of these fields is essential for the design of manufacturing processes. Other fields requiring a detailed understanding of the plasma sheath range from basic astronomical research [7] to applications mitigating the effects of the charging of artificial satellites [8].

The usual way of determining sheath potential functions is through complicated numerical models, requiring researchers to expend a great deal of effort to reproduce a model that fits their specific experimental conditions. In this letter, we discuss a very simple, highly accurate approximation to the plasma sheath potential function in the sheath. This approximation is well supported by existing numerical and experimental results and can be determined by measuring a few easily obtained experimental parameters. We also discuss a simple method for determining the magnitude of the charge on dust suspended in the plasma, based on our novel generalisation concerning the nature of that potential function.

In terrestrial plasmas, dust typically settles not in the plasma itself, but in the plasma sheath, where a balance of forces exists between, primarily, gravity and electrostatic repulsion [9]. Calculation of the charge residing on dust particles suspended in the plasma sheath has been the subject of much recent research. Several experimental techniques have been used in attempts to understand this phenomenon better. The most preva-

lent technique uses resonant oscillations of the dust, driven by various methods, and employs various assumptions about the electrical nature of the sheath to calculate the charge [10–15]. However, many of these techniques either significantly disturb the sheath, fail to account for significant intergrain forces, or both. Another technique involves observing collisions between dust particles, deriving the interaction potential, and thence the charge [16,17]. Analysis of the speed of sound in plasma crystals, dust that settles into stable, lattice-like structures under specific plasma parameters, e.g. [18], has also led to another charge calculation proposal [19].

Much of the experimental work is motivated by the lack of a coherent theory of dust charging in the plasma sheath. At least two sheath-charging theories exist in the literature [9,20], but both rely on the discredited [21] orbital motion limited (OML) charging theory, e.g. [22]. Development of such a sheath charging theory is highly desirable, as it would help to put this body of experimental work on stronger footing.

The majority of the cited experimental work relies on an *a priori* assumption that the electric field in the sheath varies linearly, or the electric potential varies parabolically, with the height above the electrode. The first purpose of this letter is to demonstrate that experimental evidence and numerical simulations available in the literature strongly support this assumption. The second purpose of this letter is to show that, once the parabolic nature of the electric potential has been suitably determined, there exists a relatively simple way of calculating the charge on dust suspended in the sheath.

Numerical support for the parabolic sheath potential has existed, unrecognised, in the literature for some time. Detailed analysis of several models have shown that, to an accuracy of better than 5% (often better than 1%) across the entire sheath, the potentials calculated by a number of accepted plasma/sheath and sheath models [9,23–30] may be modelled by simple parabolas. A novel model, similar to [9] and [29], was developed for this analysis. The chief improvements incorporated into this model were to the boundary conditions, where the log-cosine form of the plasma potential from [26] was matched to the numerical solution to allow a smooth ion velocity profile, and to the collisionality assumptions, where the new model allows for velocity-dependent variations in the ion-neutral collision cross section. Further description of this
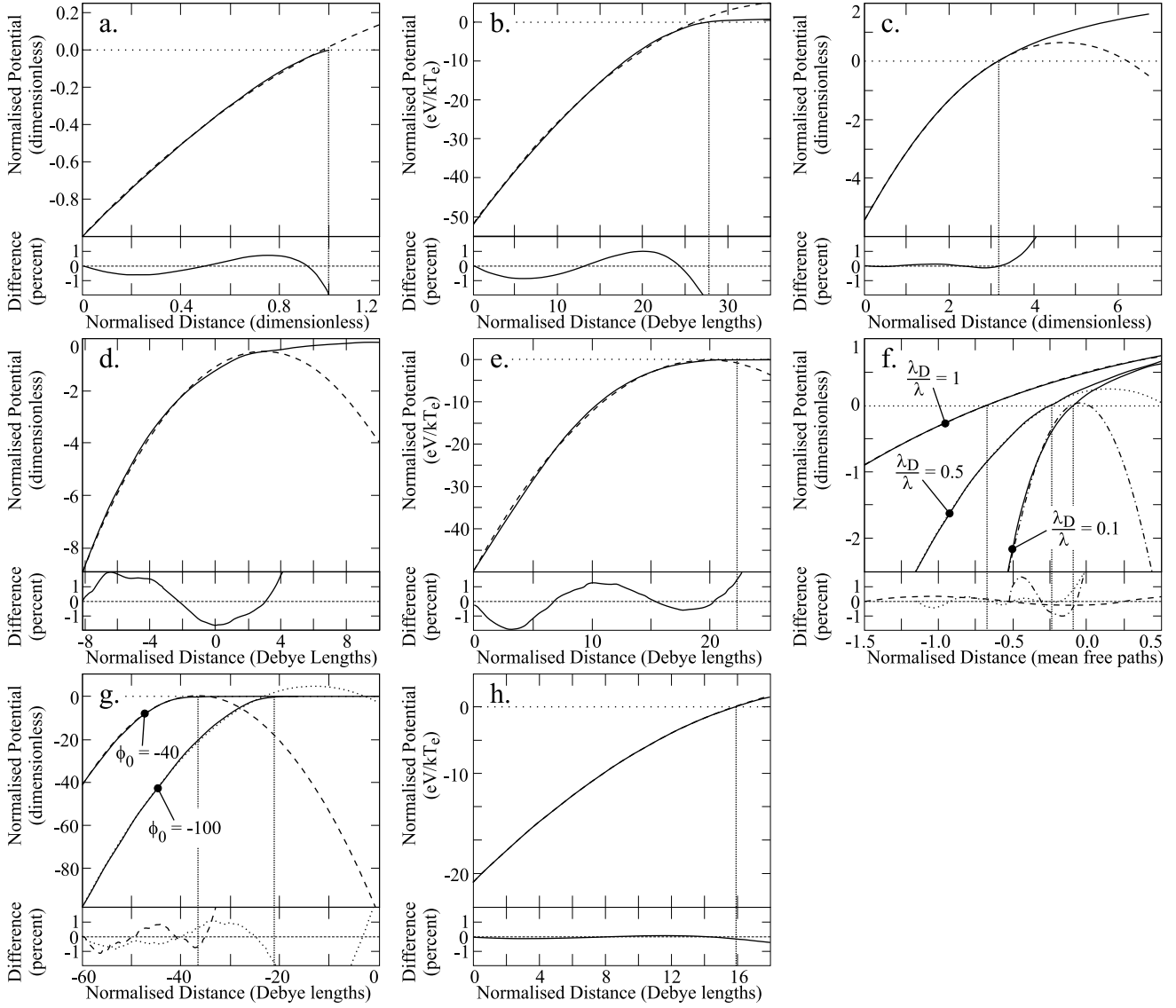
Figure 1: Parabolic Approximations to Sheath Models from the Literature. The upper portion of each frame shows the numerically derived sheath potentials from the indicated models (full curves) and their parabolic approximations (dotted/dashed/dash-dotted curves). The lower portion shows the percent difference between the parabolic approximation and the numerical results. Where applicable, the sheath edge is indicated by a vertical dotted line. The models analysed (and the applicable collisionality and generating voltage type) are as follows: a. Child-Langmuir [23,24] (collisionless DC); b. Bohm [25] (collisionless DC); c. Blank [26] (fully collisional DC); d. Franklin and Ockendon [27] (collisionless DC); e. Nitter [9] (collisional RF); f. Riemann [28] (transitional DC); g. Valentini [29] (collisionless RF); h. A model produced for the present analysis (transitional RF). The time-averaged potential is shown for the RF models. Specific normalisations, parameterisations, and sheath-edge definitions are discussed in the cited references.

model is beyond the scope of this paper, and is planned for a future article. Suffice it to say that its results were in agreement with the parabolic generalisations noted above.

The models cited above represent a cross section of plasma parameters, from collisionless to transitional to fully collisional, and encompass both DC and RF plasma generation techniques; the parabolic nature of the sheath potential appears to be a general result for this wide variety of plasma parameters in electropositive gases. Figure 1 shows the sheath potentials calculated for these models, their corresponding parabolic

fits, and the small deviations of the fits from the numerical calculation. Although the definitions of the sheath edge vary from model to model, in general, the parabolic fits are extremely good across a very substantial fraction of the sheath.

Convincing experimental evidence for a parabolic sheath electric potential also exists. Analysis of the trajectories of isolated, micron-sized dust particles injected into the plasma sheath showed them to be extremely well modelled by a damped harmonic oscillator for amplitudes approaching 20% of the sheath width. Figure 2 shows a typical dust parti-
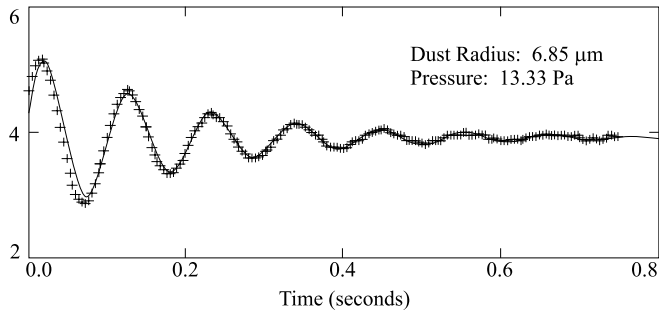
Figure 2: Typical Dust Oscillation Data [30]. The solid line is the damped harmonic theoretical curve fit to the data. Typical position measurement errors are on the order of 10 μm; these error bars are not visible at this scale.

cle trajectory, along with the fit to damped harmonic theory. Additional data and analysis has been presented elsewhere [30]. The theoretical fit breaks down for larger amplitude oscillations, most probably due to the increasing significance of charge fluctuation on the dust particle as it is exposed to different local potentials during its oscillations, however, the fits are extremely good over much larger amplitudes than would be expected from a the standard small-amplitude approximation.

These data show that the dust must oscillate in a parabolic potential energy well. The only significant forces are gravity and electrostatic repulsion [30], so the electrical potential energy is the difference between the total potential and the gravitational potential. As the total potential energy has been shown to be parabolic and the gravitational potential energy is linear, the electrical potential energy - and hence, the electric potential - must be parabolic as well. Thus, the experimental and numerical evidence both strongly support the assumption that the sheath electric potential is a parabolic function of height above the electrode.

Now that the parabolic nature of the sheath potential has been demonstrated, it remains to determine the boundary conditions for the parabola in order to get specific sheath potential functions. The conditions we feel to be convenient were the spatial location of the sheath edge, potential difference between the sheath edge and electrode as well as the electric field at the sheath edge. The sheath edge location may be determined either visually or via probe measurements, the sheath-edge-to-electrode potential difference may be determined by probe measurements, and the sheath edge electric field may be calculated from one of several proposed in the literature [26,28]. These boundary conditions together with the assumed parabolic nature of the sheath serve to specify the sheath potential function.

Knowledge of the sheath potential function leads to a method for determination of the charge on suspended dust. To do this one simply measures the mass of the dust and the height at which it suspends and then employs the force-balance equation (gravitational force equals electrostatic force) at the dust

equilibrium suspension height.

$$Q_d(r_d) = \frac{m_d(r_d) \cdot g}{E(z_{eq}(r_d))}. \tag{1}$$

Here, Q_d is the equilibrium dust charge, r_d is the dust radius, $E(z)$ is the electric field, z_{eq} is the equilibrium suspension height, m_d is the dust mass, and g is the acceleration due to gravity. As the electric field is just the negative derivative of the sheath potential, the value of the dust charge may readily be obtained.

The experimental results of [30], which show the dust charge as a function of dust radius determined by analysis of damped oscillations, may be compared with the force-balance method discussed above. These experiments were performed at two pressures, 6.67 Pa ($kT_e = 3.7$ eV and $n_0 = 1.7 \cdot 10^{15}$ m^{-3}, $z_{sh} = 9.4$ mm) and 13.33 Pa ($kT_e = 3.9$ eV and $n_0 = 2.4 \cdot 10^{15}$ m^{-3}, $z_{sh} = 8.5$ mm). Here, kT_e and n_0 are the electron temperature and electron density at the centre of the discharge determined by Langmuir probe measurements and z_{sh} is the visually determined sheath width. In all cases, the dust was monodispersive melamine formaldehyde ($\rho = 1514$ kg/m^3) suspended in an argon plasma, the electrode spacing was 40.0 mm, the RF frequency was 13.5 MHz, and the RF Amplitude was 96.4 V. The derivation of the parabolic sheath functions from these experimental conditions is presented in detail in the cited reference. It must be noted that the boundary conditions for the electric field at the sheath edge, E_s, used in [30] were merely the ones found to be the most convincing at the time that paper was written (i.e., $E_s = kT_e/e\lambda$, where e is the elementary charge, and λ is the ion mean free path); should better boundary conditions become available, they may be used with existing equilibrium height data to calculate improved values for the dust charge.

Experimental charges and the charge curve from [30] are shown in Figure 3. Also shown is a plot of the dust charge calculated from equation (1), using the linear fits to the experimental equilibrium heights $z_{eq} = -3.55 \cdot 10^2 (r_d + 9.73 \cdot 10^{-3}$ (6.67 Pa) and $z_{eq} = -3.15 \cdot 10^2 (r_d + 9.04 \cdot 10^{-3}$ (13.33 Pa), where z_{eq} and r_d are in metres. As can be seen in this figure, simple measurement of the equilibrium heights of the dust particles can provide charge values in good agreement with previous measurements and theories. We also note that the force-balance charge curves from equation (1) are remarkably similar for the two different pressures, supporting the limited dependency of the dust charge on pressure noted by other researchers [11]. The charges predicted by these curves also agree well with those from other methods [10–19], considering differences in experimental parameters.

In summary, experimental and numerical evidence both strongly support the existence of a parabolic electric potential profile in the plasma sheath. Boundary conditions specifying the exact sheath potential function are readily available from experimental measurements and theory from the literature. Once the potential function is known, the dust charge may be calculated from knowledge of the dust mass and equilibrium suspension height.
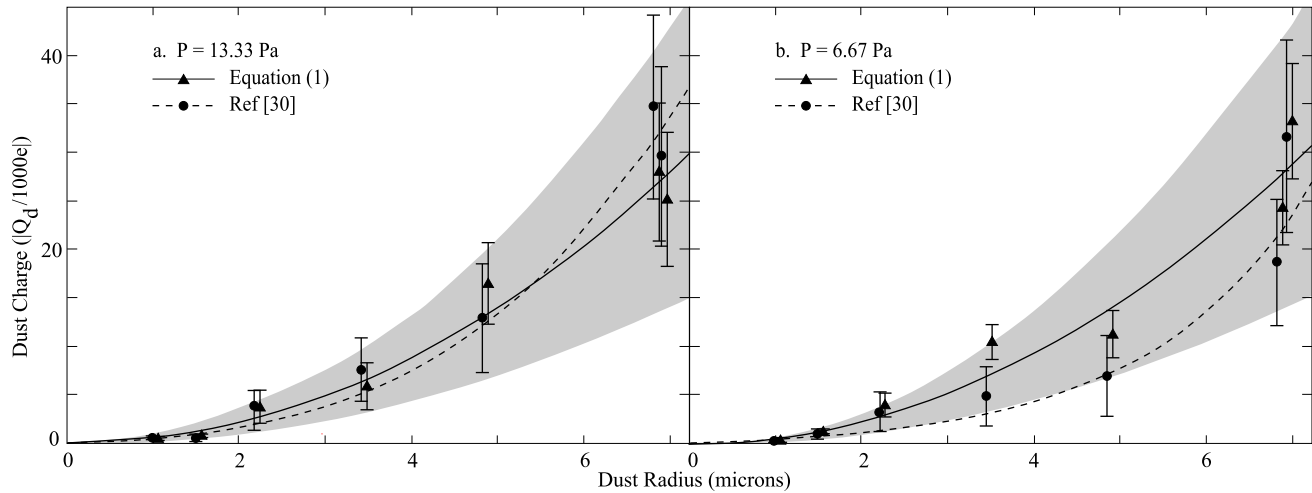
Figure 3: Variation of the Dust Charge as a Function of Dust Radius. This figure shows plots of normalised dust charge for (a) 13.33 Pa and (b) 6.67 Pa. The dashed lines with circles represent the charge curve from the theory in [30]. The solid line with triangles is a plot of the charge from equation (1), as discussed in the text. Shading indicates the region between 1.5 and 0.5 times the fit line from equation (1). The mean values for the ratios of the charges obtained from these two methods are 1.1 and 1.5 for 13.33 Pa and 6.67 Pa, respectively, indicating their good agreement. The data points have been offset from each other for clarity; the actual horizontal positions are halfway between the corresponding vertical error bars. These plots are for melamine formaldehyde suspended in an argon plasma.

*   Permanent address: Department of Physics, United States Air Force Academy, Colorado, 80840

[1] C. La Hoz, Physica Scripta **45**, 529 (1992).

[2] C. K. Goertz, Rev. Geophys. **27**, 271 (1989).

[3] G.S. Selwyn, in *International Integrated Reliability Workshop Final Report* (PUBLISHER, Lake Tahoe, California, USA, 1996), p. 122.

[4] E. Bertran et al., J. Materials Res. **13**, 2476 (1998).

[5] E. Stoffels et al., Electron Tech. **31**, 255 (1998).

[6] G. W. Kroesen et al., Eindhoven (Netherlands) Technical University Report No. BE 7328 (1998).

[7] H. Kimura and I. Mann, Astrophys. J. **499**, 454 (1998).

[8] K. Torkar et al., .

[9] T. Nitter, Plasma Sources Sci. Technol. **5**, 93 (1996).

[10] A. Melzer, T. Trottenberg, and A. Piel, Phys. Lett. A **191**, 301 (1994).

[11] T. Trottenberg, A. Melzer, and A. Piel, Plasma Sources Sci. Technol. **4**, 450 (1995).

[12] U. Kortshagen and G. Mümken, Phys. Lett. A **217**, 126 (1996).

[13] A. Homann, A. Melzer, and A. Piel, Phys. Rev. E **59**, R3835 (1999).

[14] D. A. Law et al., in *Proceedings of the 24th International Conference on Phenomena in Ionized Gases* (PUBLISHER, Warsaw, Poland, 1999), Vol. 4, p. 109.

[15] S. Nunomura, T. Misawa, N. Ohno, and S. Takamura, Phys. Rev. Lett. **83**, 1970 (1999).

[16] U. Konopka, L. Ratke, and H. M. Thomas, Phys. Rev. Lett. **79**, 1269 (1997).

[17] U. Konopka, G.E. Morfill, and L. Ratke, Phys. Rev. Lett. **84**, 891 (2000).

[18] G. E. Morfill, H. M. Thomas, U. Konopka, and M. Zuzic, Phys. Plasmas **6**, 1769 (1999).

[19] D. Samsonov, J. Goree, Z. W. Ma, and A. Bhattacharjee, Phys. Rev. Lett. **83**, 3649 (1999).

[20] X. Chen, IEEE Trans. Plasma Sci. **25**, 1117 (1997).

[21] J. E. Allen, B. M. Annaratone, and U. de Angelis, J. Plasma Phys. **63**, 299 (2000).

[22] J. E. Allen, Physica Scripta **45**, 497 (1992).

[23] C. D. Child, Phys. Rev. **32**, 492 (1911).

[24] I. Langmuir, Phys. Rev. **2**, 450 (1913).

[25] D. Bohm, in *The Characteristics of Electrical Discharges in Magnetic Fields*, edited by A. Guthrie and R.K. Wakerling (McGraw-Hill, New York, ADDRESS, 1949), Chap. 3.

[26] J. L. Blank, Phys. Fluids **11**, 1686 (1968).

[27] R. N. Franklin and J. R. Ockendon, J. Plasma Phys. **4**, 371 (1970).

[28] K. U. Riemann, Phys. Plasmas **4**, 4158 (1997).

[29] H. B. Valentini, J. Appl. Phys. **86**, 6665 (1999).

[30] E. B. Tomme, B. M. Annaratone, and J. E. Allen, Plasma Sources Sci. Technol. **9**, (2000).